2016 SYDNEY BOYS HIGH SCHOOL
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided with this paper
- Leave your answers in the simplest exact form, unless otherwise stated
- All necessary working should be shown in every question if full marks are to be awarded
- Marks may **NOT** be awarded for messy or badly arranged work
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 3–6

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 8–19

90 marks

- Attempt Questions 11–16
- Allow about 2 hour and 45 minutes for this section

Examiner: B.K.

Section I

10 marks
Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. $\cos\left(\frac{-5\pi}{4}\right)$ is the same as

(A) $-\cos\left(\frac{\pi}{4}\right)$

(B) $-\cos\left(\frac{5\pi}{4}\right)$

(C) $\cos\left(\frac{-\pi}{4}\right)$

(D) $\cos\left(\frac{\pi}{4}\right)$

2. What is the domain and range of the function $y = \frac{1}{\sqrt{x-9}}$?

(A) $x \geq 9$ and $y > 0$

(B) $x > 9$ and $y > 0$

(C) $-\infty \leq x \leq \infty$ and $-\infty \leq y \leq \infty$

(D) $-3 \leq x \leq 3$ and $y < 0$

3. Evaluate $\lim\limits_{x \to -4} \frac{x^2 + 4x}{x + 4}$

(A) Does not exist

(B) $-\frac{1}{4}$

(C) 4

(D) -4

4. What is the area bounded by the curve $y = 3\sin 2x$ and the x-axis between $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$?

(A) $\left| \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 3\sin 2x \, dx \right|$

(B) $-\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 3\sin 2x \, dx$

(C) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 3\sin 2x \, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} 3\sin 2x \, dx$

(D) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 3\sin 2x \, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} 3\sin 2x \, dx$

5. The derivative of $e^{\sin x}$ is equal to

(A) $(\cos x)e^{\sin x}$

(B) $e^{\cos x}$

(C) $e^{\sin x}$

(D) $(\cos x)e^{\cos x}$

6. A primitive of $e^{3x} + \sin(3x)$ is

(A) $e^{3x} - \dfrac{\cos(3x)}{3}$

(B) $\dfrac{e^{3x}}{3} - \dfrac{\cos(3x)}{3}$

(C) $3e^{x} + 3\cos(3x)$

(D) $\dfrac{e^{3x}}{3} - \cos(3x)$

7. Fifty tickets are sold in a raffle. There are two prizes. Michelle buys 5 tickets. The probability that she does not win either prize is given by

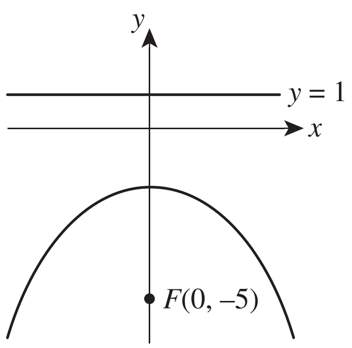
(A) $1-\dfrac{5}{50}\times\dfrac{4}{49}$

(B) $\dfrac{45}{50}+\dfrac{44}{49}$

(C) $\dfrac{45}{50}\times\dfrac{44}{50}$

(D) $\dfrac{45}{50}\times\dfrac{44}{49}$

8. A parabola is shown below



What is the equation of the parabola with directrix $y = 1$ and focus $F(0, –5)$

(A) $x^2 = 12(y + 2)$

(B) $x^2 = 12(y + 5)$

(C) $x^2 = –12(y + 2)$

(D) $x^2 = –24(y + 5)$

9. $\dfrac{\log_5 125}{\log_5 5}$ simplifies to

(A) $\log_5 25$

(B) $\log_5 120$

(C) 25

(D) 3

10. Let $a = e^x$. Which expression is equal to $\log_e(a^2)$?

(A) e^{2x}

(B) e^{x^2}

(C) $2x$

(D) x^2

Section II

90 marks
Attempt Questions 11–16

Allow about 2 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Use a SEPARATE writing booklet

a) Differentiate with respect to x

(i) $3x^e$ **(1)**

(ii) $\log_e(\tan x)$ **(1)**

b) If $y = 10x^2 + x - 2$ has roots α and β, find

(i) $\alpha + \beta$ **(1)**

(ii) $\alpha^2 + \beta^2$ **(2)**

c) The volume V cm^3 of unmelted ice-cream in a container, t seconds after it has been removed from a freezer (at $0°$) is modelled by the equation **(2)**

$$V(t) = 0.02t^2 - 4t + 200.$$

Find the rate (in cm^3/sec) at which the ice-cream is melting 40 seconds after it is removed from the freezer.

d) Solve simultaneously **(2)**

$$a + b = -2$$
$$2a + b = 0$$

Question 11 continues on page 9

Question 11 (continued)

e) Find the equation of the perpendicular bisector of the interval joining (6, 8) and (0, –4). **(2)**

f) Factorise fully $16x^3 - 54$ **(2)**

g) The graph of $y = f(x)$ passes through the point (2, 65) and $f'(x) = 12x + 29$. **(2)**

Find $f(x)$.

End of Question 11

Question 12 (15 Marks) Use a SEPARATE writing booklet

a) Find $\displaystyle\int (\sin 2x + e^{-3x})\, dx$ **(2)**

b) Evaluate $\displaystyle\int_1^5 \left(2 + \frac{1}{x}\right)^2 dx$ **(2)**

c) The table shows the values of $f(x)$ for five values of x.

x	1	1.5	2	2.5	3
y	5	1	–2	3	7

Use Simpson's Rule with these five values to estimate $\displaystyle\int_1^3 f(x)\, dx$ **(2)**

d) Solve for x: $\log_5(2x+1) - \log_5 x = 2$ **(2)**

e) A chemical factory releases polluted water into a holding pond in periods of 30 seconds. The rate of change of the total volume of polluted water which has been released after time t seconds from the start of the period is given by **(2)**

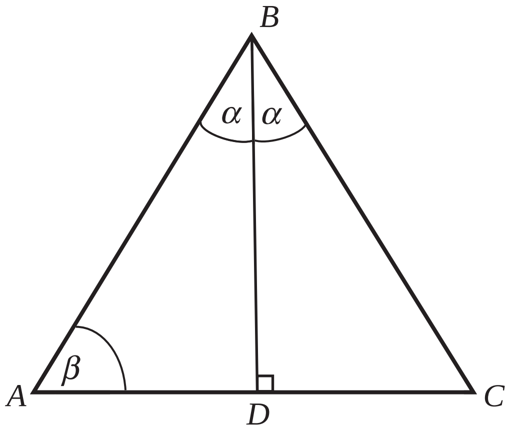
$$30t - t^2 \text{ cm}^3/\text{s} \text{ for } 0 \le t \le 30 .$$

Find the total volume of polluted water released for such a 30 second period.

Question 12 continues on page 11

Question 12 (continued)

f) The triangle ABC is isosceles with $AB = BC$.
Let $\angle ABD = \angle CBD = \alpha$ and $\angle BAD = \beta$ as shown below



(i) Show $\sin\beta = \cos\alpha$ **(1)**

(ii) By applying the sine rule in ΔABC, show that **(2)**

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

(iii) From (ii), and given that $0 < \alpha < \dfrac{\pi}{4}$, hence show that the limiting sum of the following geometric series **(2)**

$$\sin 2\alpha + \sin 2\alpha\cos^2\alpha + \sin 2\alpha\cos^4\alpha + \sin 2\alpha\cos^6\alpha + ...$$

is equal to $2\cot\alpha$

End of Question 12

Question 13 (15 Marks) Use a SEPARATE writing booklet

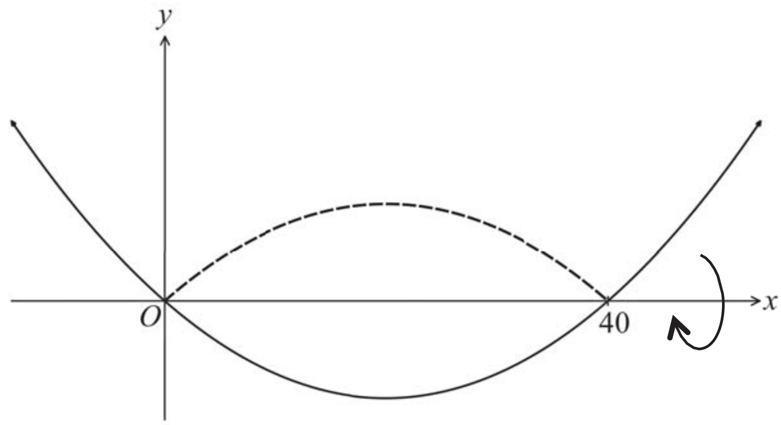
a) Find the equation of the tangent to the curve **(2)**

$$y = 2\log_e(3x-2)$$

at the point (1, 0).

b) A petrol tank is designed by rotating the curve $y = \frac{1}{5}x(x-40)$ about the x-axis between $x = 0$ and $x = 40$. **(2)**

If units are in centimetres, how many litres would the tank hold?



c) The displacement from O of a particle travelling in a straight line is given by $x = 2\sin\frac{\pi}{3}t$, where x is in cm and t is in seconds.

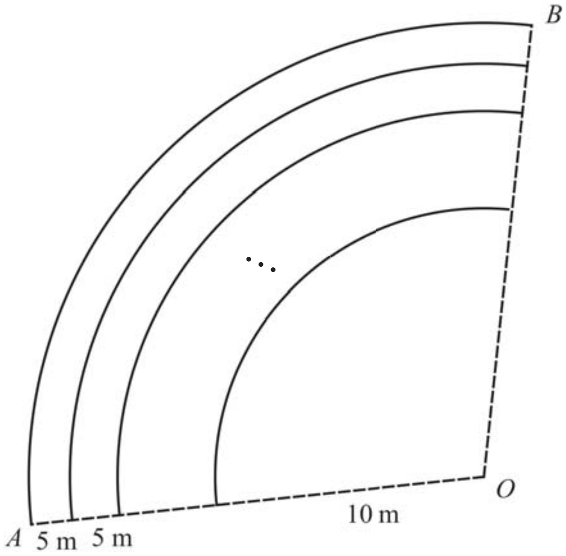
(i) Find the first time when the particle is at rest. **(2)**

(ii) Find the distance travelled in the first 4 seconds, correct to 1 decimal place. **(2)**

Question 13 continues on page 13

Question 13 (continued)

d) Jacqueline’s farming property is in the shape of a sector.
She planted pine trees in rows along arcs. The first row started 10 m from the point O.
There were 13 trees in the first row, 19 trees in the second row, 25 trees in the third row and so on.
Each row is planted 5 m from the previous row.



(i) If Jacqueline planted 6525 pine trees in total, how many rows did she plant? **(2)**

(ii) If $\angle AOB$ is 1 radian, what is the length of the last planted row? **(2)**

e) Sketch the curve $y = xe^{\frac{x}{2}}$ showing any asymptotes and stationary points. **(3)**
It is NOT necessary to find any points of inflexion.

End of Question 13

Question 14 (15 Marks) Use a SEPARATE writing booklet

a) In a game, two players take turns at drawing and then immediately replacing a marble from a bag. The bag contains 2 green and 3 red marbles.
Player A draws first.
For A to win he must draw a green marble.
For B to win he must draw a red marble. Find the probability that

(i) A wins on his first draw. **(1)**

(ii) B wins on his first draw. **(1)**

(iii) A wins in fewer than 4 of his turns. **(2)**

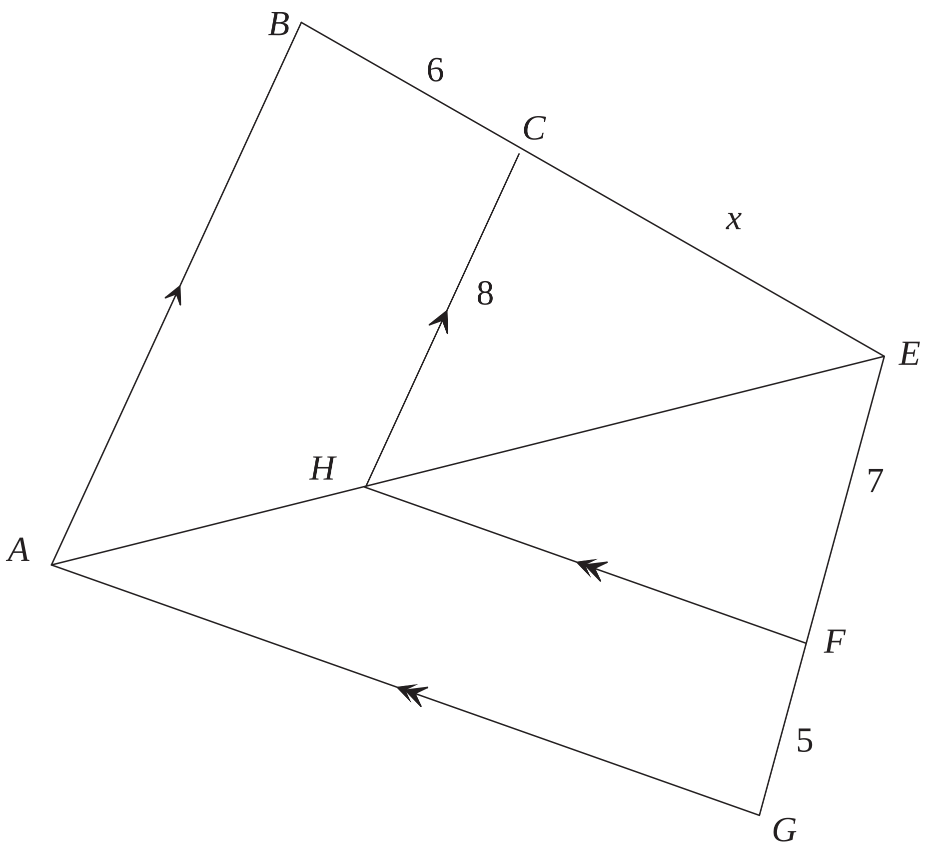
(iv) A wins eventually. **(2)**

b) Find any points of inflexion on the curve $y = x^3 + x^2$ **(2)**

Question 14 continues on page 15

Question 14 (continued)

c) In the diagram $FH \parallel GA$ and $CH \parallel BA$, $BC = 6$, $CE = x$, $EF = 7$ and $FG = 5$.



(i) Find the value of x, giving reasons. **(2)**

(ii) Find the length of BA, without giving reasons. **(1)**

d) Exactly 12 years ago, Paul took out a mortgage of \$500 000 to buy a house.
The loan was taken over 25 years at 12% p.a. with interest compounding monthly and Paul makes monthly repayments.
Paul has just won a lottery prize of \$400 000.

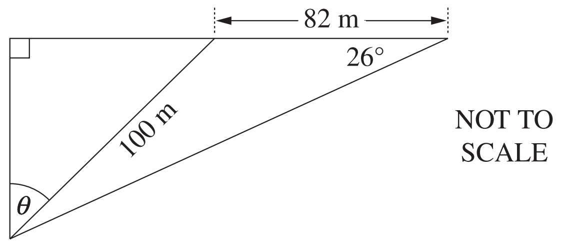
(i) Show that the prize is insufficient to pay out the remaining debt. **(3)**

(ii) How many payments will still be required to pay off the debt? **(1)**
(You may assume that Paul puts the entire prize into paying off the debt.)

End of Question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet

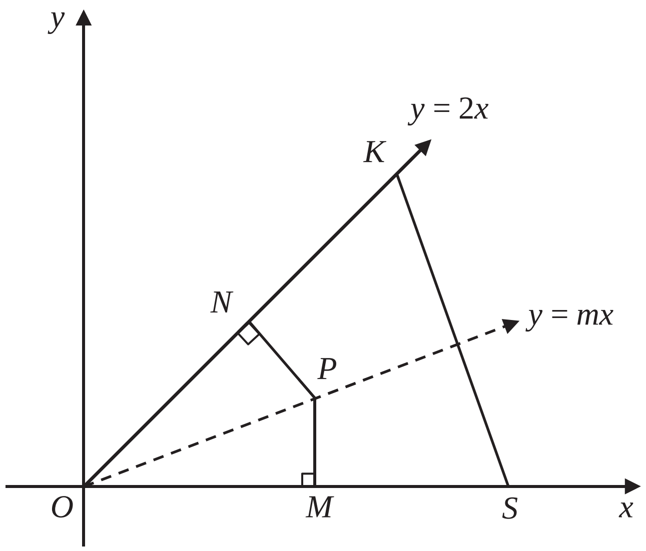
a) What is the value of θ, to the nearest degree? **(2)**



b) Consider the function $f(x) = kx^2 - (3k-4)x + k$. **(2)**

Show that $f(x)$ is positive definite for $\frac{4}{5} < k < 4$.

c) In the diagram below $P(a, ma)$ is a point on the line $y = mx$ which is the internal bisector of $\angle KOS$.
The line KO is $y = 2x$.



(i) Using the fact that the perpendicular distances from P to OK and OS are equal, show that $\dfrac{|(2-m)a|}{\sqrt{5}} = ma$ **(2)**

(ii) Hence show that $m = \dfrac{2}{1+\sqrt{5}}$ **(1)**

(iii) Using this result, find the co-ordinates of P if $PN = \sqrt{5} - 1$ **(2)**

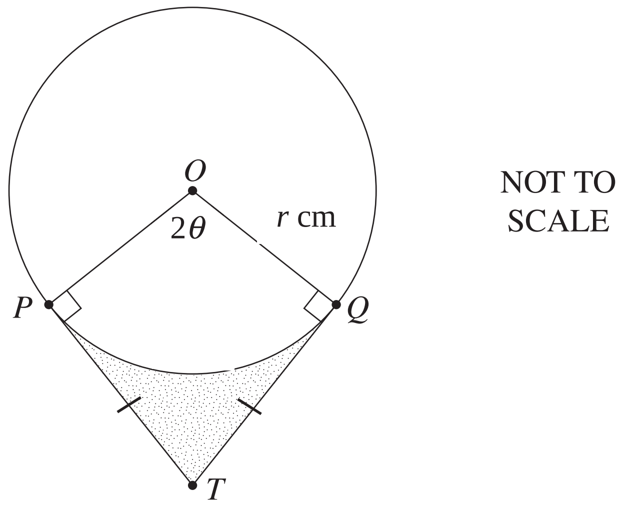
Question 15 continues on page 17

Question 15 (continued)

d) (i) Find $\frac{d}{dx}(x^2+1)^3$. **(1)**

(ii) Hence evaluate $\int_0^1 5x(x^2+1)^2\,dx$ **(2)**

e) P and Q are points on a circle of radius r, and the chord PQ subtends an angle of 2θ radians at its centre O.



A is the shaded area enclosed by the minor arc PQ and the tangents to the circle at P and Q from an external point T.

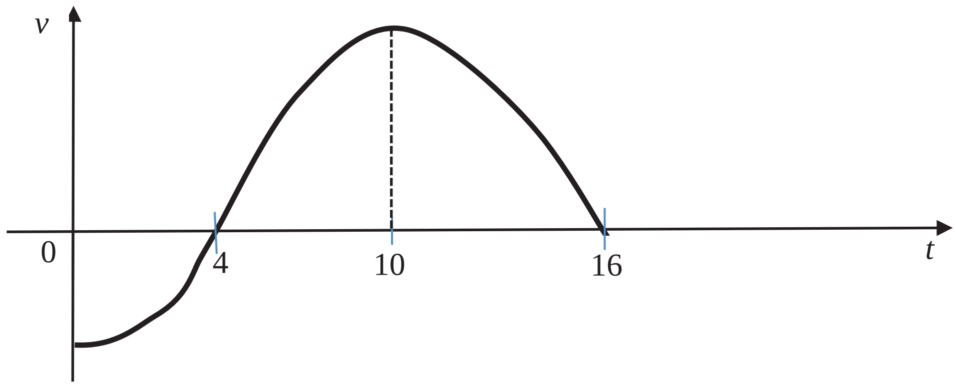
(i) Show that $PT = r\tan\theta$. **(1)**

(ii) Find an expression for the shaded area, A, in terms of r and θ. **(2)**

End of Question 15

Question 16 (15 Marks) Use a SEPARATE writing booklet

a) The following is a velocity / time graph of a particle for $0 \leq t \leq 16$.



(i) When is the particle moving to the right? **(1)**

(ii) When is the acceleration of the particle positive? **(1)**

(iii) When is the particle furthest from its starting point? **(1)**

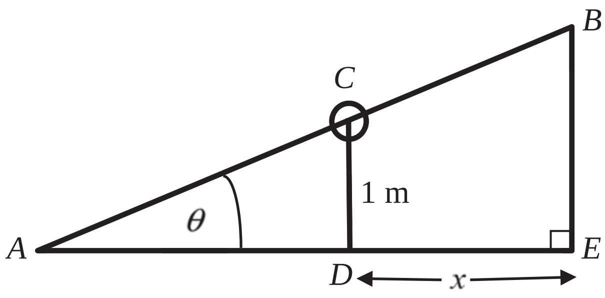
(iv) Approximately when does it return to its starting point? **(1)**

(v) Sketch the graph of the particle's acceleration against time. **(2)**

Question 16 continues on page 18

Question 16 (continued)

b) The diagram below shows a straight rod AB of length 8 m hinged to the ground at A.
CD is a rod of 1m.
The end C is free to slide along AB while the end is touching the ground floor such that CD is perpendicular to the ground.



Let $DE = x$ m and $\angle BAE = \theta$, where $\frac{1}{8} < \sin\theta < 1$.

(i) Express x in terms of θ. **(2)**

(ii) Find the maximum value of x as θ varies. **(2)**

(iii) Let M be the area of trapezium $CDEB$. Show that **(2)**

$$M = \left(\frac{1+8\sin\theta}{2}\right)(8\cos\theta - \cot\theta)$$

(iv) Does M attain a maximum when x reaches its maximum? **(3)**
Justify your answer.

End of paper

2016 SYDNEY BOYS HIGH SCHOOL
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Mathematics

Sample Solutions

Question	Teacher
Q11	AMG
Q12	BK
Q13	JWC
Q14	RD
Q15	JM
Q16	RB

MC Answers

1.	A	**3.**	D	**5.**	A	**7.**	D	**9.**	D
2.	B	**4.**	D	**6.**	B	**8.**	C	**10.**	C

1. $\cos\left(\frac{-5\pi}{4}\right)$ is the same as

(A) $-\cos\left(\frac{\pi}{4}\right)$

(B) $-\cos\left(\frac{5\pi}{4}\right)$

(C) $\cos\left(\frac{-\pi}{4}\right)$

(D) $\cos\left(\frac{\pi}{4}\right)$

$$\begin{aligned}\cos\left(\frac{-5\pi}{4}\right) &= \cos\left(\frac{5\pi}{4}\right) && [\cos x \text{ is even}]\\ &= \cos\left(\pi+\frac{\pi}{4}\right)\\ &= -\cos\left(\frac{\pi}{4}\right)\end{aligned}$$

2. What is the domain and range of the function $y=\frac{1}{\sqrt{x-9}}$?

(A) $x \geq 9$ and $y > 0$

(B) $x > 9$ and $y > 0$

(C) $-\infty \leq x \leq \infty$ and $-\infty \leq y \leq \infty$

(D) $-3 \leq x \leq 3$ and $y < 0$

$$\begin{aligned}&x-9>0\\ &\therefore x>9\\ &y=\frac{1}{\sqrt{x-9}}>0\end{aligned}$$

3. Evaluate $\lim\limits_{x \to -4} \dfrac{x^2 + 4x}{x + 4}$

(A) Does not exist

(B) $-\dfrac{1}{4}$

(C) 4

(D) -4

$$\begin{aligned}\lim_{x \to -4} \frac{x^2 + 4x}{x + 4} &= \lim_{x \to -4} \frac{x(x+4)}{x+4} \\ &= \lim_{x \to -4} x \\ &= -4\end{aligned}$$

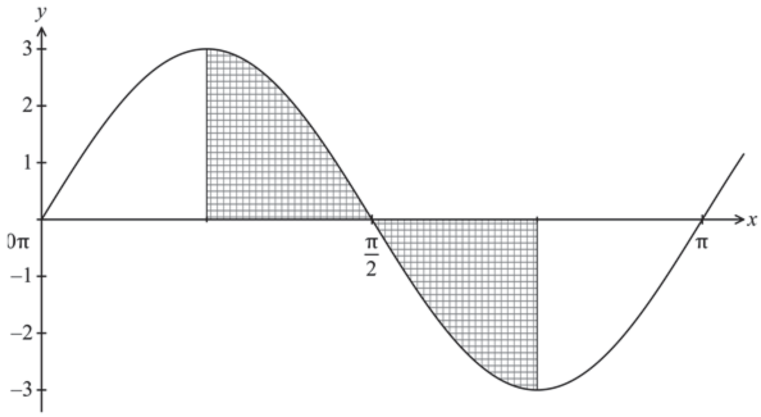
4. What is the area bounded by the curve $y = 3\sin 2x$ and the x-axis between $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$?

(A) $\left| \displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 3\sin 2x \, dx \right|$

(B) $-\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 3\sin 2x \, dx$

(C) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 3\sin 2x \, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} 3\sin 2x \, dx$

(D) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 3\sin 2x \, dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} 3\sin 2x \, dx$



5. The derivative of $e^{\sin x}$ is equal to

(A) $(\cos x)e^{\sin x}$

(B) $e^{\cos x}$

(C) $e^{\sin x}$

(D) $(\cos x)e^{\cos x}$

$$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$$

6. A primitive of $e^{3x} + \sin(3x)$ is

(A) $e^{3x} - \dfrac{\cos(3x)}{3}$

(B) $\dfrac{e^{3x}}{3} - \dfrac{\cos(3x)}{3}$

(C) $3e^{x} + 3\cos(3x)$

(D) $\dfrac{e^{3x}}{3} - \cos(3x)$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C \text{ and } \int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$

7. Fifty tickets are sold in a raffle. There are two prizes. Michelle buys 5 tickets. The probability that she does not win either prize is given by

(A) $1-\frac{5}{50}\times\frac{4}{49}$

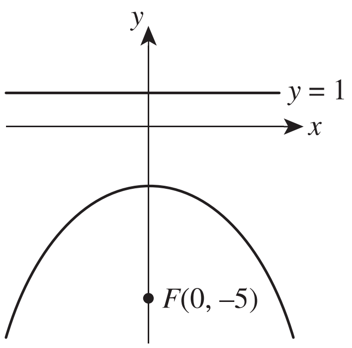
(B) $\frac{45}{50}+\frac{44}{49}$

(C) $\frac{45}{50}\times\frac{44}{50}$

(D) $\frac{45}{50}\times\frac{44}{49}$

A is wrong as it allows Michelle to win one of the prizes
B is wrong due to the addition
C is wrong as it assumes the ticket is replaced before drawing the next prize.

8. A parabola is shown below



What is the equation of the parabola with directrix $y = 1$ and focus $F(0, -5)$

(A) $x^2 = 12(y + 2)$

(B) $x^2 = 12(y + 5)$

(C) $x^2 = -12(y + 2)$

(D) $x^2 = -24(y + 5)$

The parabola is concave down and the vertex is the midpoint of (0, 1) and (0, –5) i.e. (0, –2).
a = focal distance = 3.

9. $\dfrac{\log_5 125}{\log_5 5}$ simplifies to

(A) $\log_5 25$

(B) $\log_5 120$

(C) 25

(D) 3

$$\frac{\log_5 125}{\log_5 5} = \frac{\log_5 5^3}{1} = 3\log_5 5 = 3$$

10. Let $a = e^x$. Which expression is equal to $\log_e(a^2)$?

(A) e^{2x}

(B) e^{x^2}

(C) $2x$

(D) x^2

$$\begin{aligned}\log_e(a^2) &= 2\log_e a \\ &= 2\log_e e^x \\ &= 2x\log_e e \\ &= 2x\end{aligned}$$

2U THSC 2016 Multiple choice solutions

Mean (out of 10): 8.82

1. $\cos\left(-\frac{5\pi}{4}\right)$

$= \cos\left(\frac{5\pi}{4}\right)$

$= \cos\left(\pi + \frac{\pi}{4}\right)$

$= -\cos\frac{\pi}{4}$ (A)

A	166
B	9
C	3
D	6

2. $y = \frac{1}{\sqrt{x-9}}$ $\quad x > 9$

$y > 0$ (B)

A	14
B	171
C	0
D	0

3. $\lim_{x \to -4} \frac{x^2+4x}{x+4}$

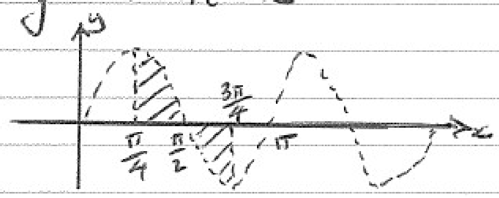
$= \lim_{x \to -4} \frac{x(x+4)}{x+4}$

$= \lim_{x \to -4} x$

$= -4$ (D)

A	10
B	1
C	8
D	166

4. $y = 3\sin 2x$



$\text{Area} = \int_{\frac{\pi}{4}}^{\pi/2} 3\sin 2x\,dx - \int_{\pi/2}^{\frac{3\pi}{4}} 3\sin 2x\,dx$ (D)

A	20
B	0
C	19
D	146

5. $\frac{d}{dx}\left(e^{\sin x}\right)$

$= e^{\sin x} \cdot \cos x$ (A)

A	182
B	0
C	1
D	2

6. $\int \left(e^{3x} + \sin 3x\right) dx$

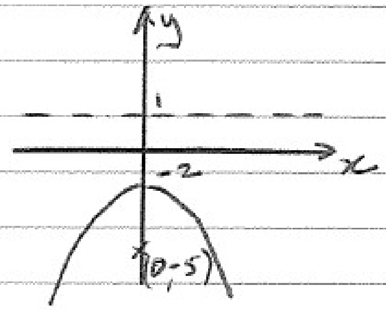
$= \frac{1}{3}e^{3x} - \frac{1}{3}\cos 3x + C$ (B)

A	0
B	178
C	3
D	4

7. $\frac{45}{50} \times \frac{44}{49}$ (D)

A	29
B	4
C	3
D	149

8.



$$x^2 = -4 \times 3 \times (y+2)$$

$$x^2 = -12(y+2)$$

C

A	5
B	5
C	169
D	5

9. $\dfrac{\log_5 125}{\log_5 5}$

$= \log_5 125$

$= \log_5 5^3$

$= 3$

D

A	5
B	3
C	7
D	170

10. $\log_e(e^{2x})$

$= \log_e (e^x)^2$

$= 2x$

C

A	6
B	4
C	162
D	13

Solutions: SBHS Maths THSC 2016

Question 11

(a) (i) $\dfrac{d}{dx}\left(3x^{e}\right) = 3ex^{e-1}$

[Comment: The unusual nature of this function confused many candidates.]

(ii) $\dfrac{d}{dx}\ln\left(\tan x\right) = \dfrac{\sec^2 x}{\tan x}$

$= \cot x + \tan x$

$= \sec x \operatorname{cosec} x$

[Comment: Most candidates succeeded to find this derivative. Many made correct, but unnecessary, simplifications.]

(b) $y = 10x^2 + x - 2$

$a = 10,\ \ b = 1,\ \ c = -2$

(i) $\alpha + \beta = \dfrac{-b}{a}$

$= -\dfrac{1}{10}$

(ii) $\alpha^2 + \beta^2 = \left(\alpha + \beta\right)^2 - 2\alpha\beta$

$= \left(-\dfrac{1}{10}\right)^2 - 2\left(-\dfrac{2}{10}\right)$

$= \dfrac{41}{100}$

[Comment: Most had no difficulty with part (i), but many misremembered the formula in part (ii).]

(c) $V = 0.02t^2 - 4t + 20 - 200$

$\dfrac{dV}{dt} = 0.04t - 4$

After 40 seconds:

$\dfrac{dV}{dt} = 0.04 \times 40 - 4$

$= -2.4$

Thus melting at 2.4cm/sec

[Comment: Very well answered.]

(d) $a+b=-2$ --(1)

$2a+b=0$ $--(2)$

$a=2$ (2)-(1)

Clearly, in (1), b = -4

Solution (2,-4)

[Comment: Usually well answered. Some subtracted incorrectly.]

(e) Perpendicular bisector of the interval joining (6,8) to (0,-4).

Mid-Point $M\left(\frac{6+0}{2},\frac{8-4}{2}\right)=M(3,2)$

Gradient $m=\frac{8+4}{6-0}$

$=2$

Thus $m_{\perp}=-\frac{1}{2}$

$\therefore$ Line is $y-2=-\frac{1}{2}(x-3)$

$x+2y-7=0$

[Comment: Generally well answered. Some failed to find the mid-point, while others managed to get the gradient wrong. Many did not state the result in general form, but did not lose a mark.]

(f) $16x^3-54=2\left(8x^3-27\right)$

$=2(2x-3)\left(4x^2+6x+9\right)$

[Comment: Generally well answered, but those who had fractions or irrationals in their factors lost some, or both, marks.]

(g) Given $y'=12x+29$

Thus $y=6x^2+29x+C$

When $x=2,\ y=65$

Thus $65=6\times4+58-C$

$C=-17$

Hence $y=6x^2+29x-17$

[Comment: Very well answered.]

Q12 (a) $\int(\sin 2x + e^{-3x})\,dx$

Done well.

$= \dfrac{-\cos 2x}{2} - \dfrac{e^{-3x}}{3} + C$

2

(b) $\int_1^5 \left(2+\frac{1}{x}\right)^2 dx$

Mostly done well. Some did not expand first. The other error was in not being able to integrate 1/x

$= \int_1^5 \left(4 + \frac{4}{x} + \frac{1}{x^2}\right) dx$

$= \left[4x + 4\ln x - \frac{1}{x}\right]_1^5$

$= \left[\left(20 + 4\ln 5 - \frac{1}{5}\right) - (4 + 4\ln 1 - 1)\right]$

$= 16\frac{4}{5} + 4\ln 5$

2

(c) $\int_1^3 f(x)\,dx = \frac{w}{3}\left[\text{Ends} + 4(\text{odds}) + 2(\text{evens})\right]$

$\doteqdot \frac{0.5}{3}\left[5 + 7 + 4(1+3) + 2(-2)\right]$

$\doteqdot 4$

2

Done well if they knew the formula. Some used Trap Rule.

(d) $\log_5(2x+1) - \log_5 x = 2$

$\log_5\left(\frac{2x+1}{x}\right) = 2$

$\frac{2x+1}{x} = 25$

$2x + 1 = 25x$

$23x = 1 \Rightarrow x = \frac{1}{23}$

2

Many did not know log rules so could not solve equation.

12 (e) $\frac{dV}{dt} = 30t - t^2$

$V = \int_0^{30} (30t - t^2)\,dt$

$= \left[15t^2 - \frac{t^3}{3}\right]_0^{30}$

$= \left[(13500 - 9000) - 0\right]$

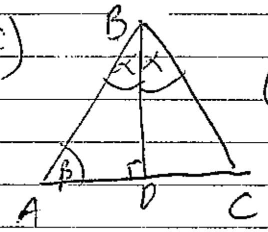
$= 4500\ \text{cm}^3$

ie $4500\ \text{cm}^3$ released.

2

Done well.

(f)



(i) $\sin\beta = \frac{BD}{AB}$

$\cos\alpha = \frac{BD}{AB}$

$\therefore \sin\beta = \cos\alpha$

Done well.

(ii) By sine rule, in $\triangle ABC$,

$\frac{\sin 2\alpha}{AC} = \frac{\sin\beta}{BC}$

$\Rightarrow \frac{\sin 2\alpha}{2\times AD} = \frac{\cos\alpha}{AB}$ (since $\sin\beta = \cos\alpha$ and $AB = BC$

$\frac{\sin 2\alpha}{2} = \cos\alpha \times \frac{AD}{AB}$

12(f)(cont)(ii)

2

$\Rightarrow \sin 2\alpha = 2\cos\alpha \sin\alpha$ # $\left(\sin\alpha = \frac{AD}{AB}\right)$

Many did not see the connection between AC and 2 AD so could not proceed meaningfully.

(iii) $S_\infty = \frac{a}{1-r}$

$a = \sin 2\alpha$, $r = \cos^2\alpha$

$\Rightarrow S_\infty = \frac{\sin 2\alpha}{1-\cos^2\alpha}$ ✓

$= \frac{2\sin\alpha\cos\alpha}{\sin^2\alpha}$

$= \frac{2\cos\alpha}{\sin\alpha}$ ✓ 2

$= 2\cot\alpha$.

This question was done well.

13a)

$y = 2\ln(3x-2)$

$y' = 2\times\left(\frac{3}{3x-2}\right)$

At $x = 1$,

$m = 2\times\left(\frac{3}{3-2}\right) = 6$ ✓

$\therefore y - 0 = 6(x-1)$

$\therefore y = 6x - 6$ ✓

Aw 1 : gradient

Aw 2 : correct eqn. of the tangent

13b)

$y = \frac{x^2}{5} - 8x$

Some candidates had great difficulty finding y^2.

$y^2 = \frac{x^4}{25} - \frac{16x^3}{5} + 64x^2$

$V = \pi \int_0^{40} \frac{x^4}{25} - \frac{16x^3}{5} + 64x^2 \, dx$

$= \pi \left[\frac{x^5}{125} - \frac{16x^4}{20} + \frac{64x^3}{3} \right]_0^{40}$ ✓

$= \pi (819200 - 2048000 + 136533\frac{1}{3})$

$= 136533\frac{\pi}{3} \text{ cm}^2$

$-\frac{1}{2}$ if not converted correctly to litres.

$= \frac{2048\pi}{15}$ Litres

≈ 428.932117 litres ✓

Q13c

i) $x = 2\sin\frac{\pi}{3}t$

$V = \frac{dx}{dt} = \frac{2\pi}{3}\cos\frac{\pi t}{3}$

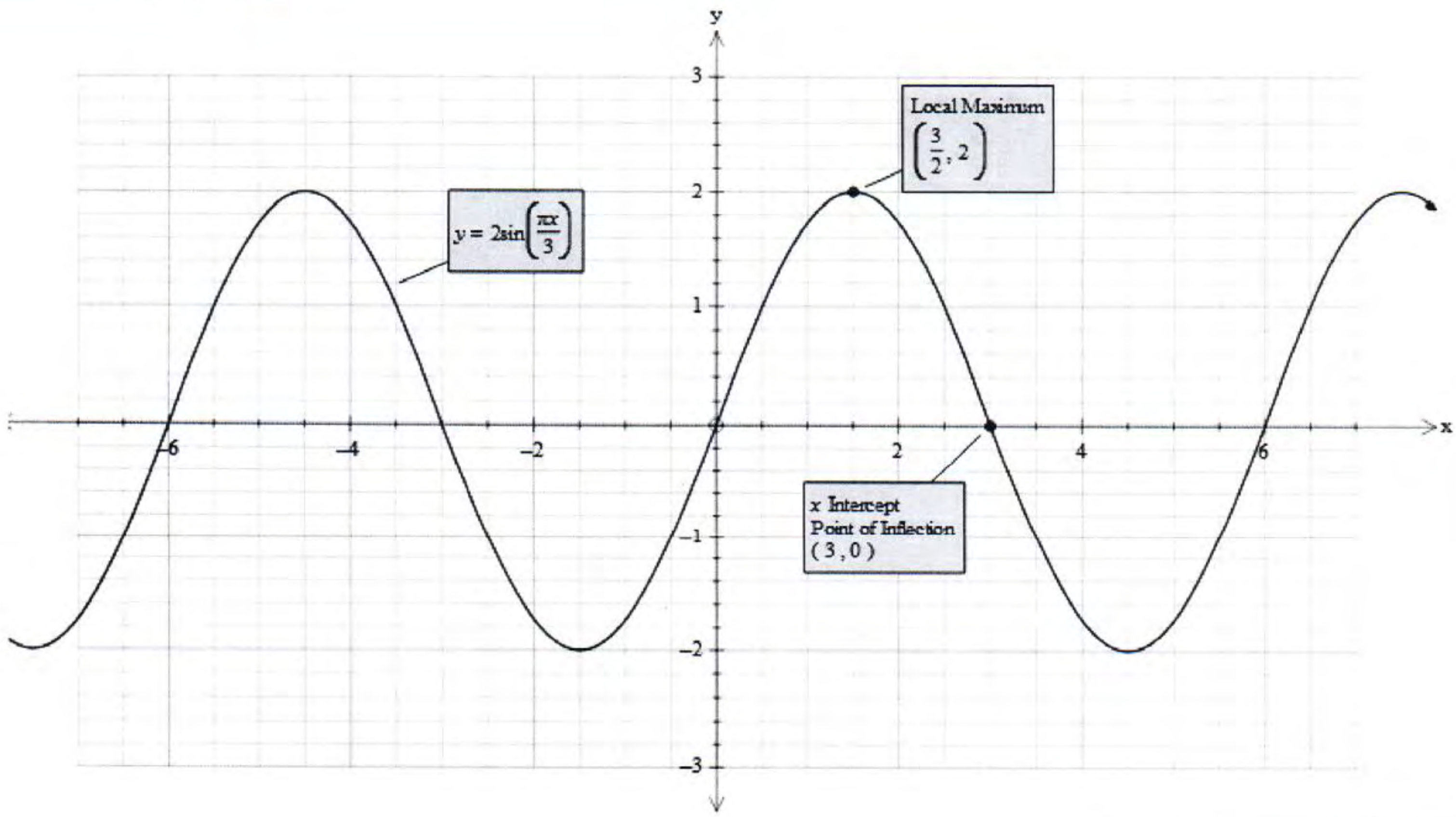
$V=0, \quad \cos\frac{\pi t}{3} = 0$ ✓

$\frac{\pi t}{3} = \frac{\pi}{2}$

$t = \frac{3}{2}$ ✓

∴ The particle is first at rest at $3/2$ seconds

ii)



t	0	$1\frac{1}{2}$	3	4
d	0	2	0	$-\sqrt{3}$

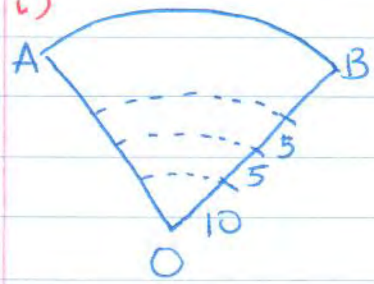
✓ or use graph

∴ Distance travelled in the first 4 sec is $4+\sqrt{3}$ ✓

AW 1 $4-\sqrt{3}$
$2+\sqrt{3}$

Q13 d)

i)



Generally well done.

i) 13, 19, 25, ...

$6525 = \frac{n}{2}(26 + (n-1)6)$

$6n^2 + 20n - 13050 = 0$ ✓

$3n^2 + 10n - 6525 = 0$

$n = \frac{-10 \pm 280}{6}$

$n > 0 \therefore n = 45$ ✓

ii)

$l = r\theta$

$l = (44 \times 5 + 10) \times 1\,rad$ ✓

$= 230\,cm$ ✓

AW 1 : $45 \times 5 + 10$

or 45×5

AW 2 : 230 cm

Some candidate did not interpret question correctly and found sum of arc lengths.

Q13e)

Intercepts $x=0$, $y=0$

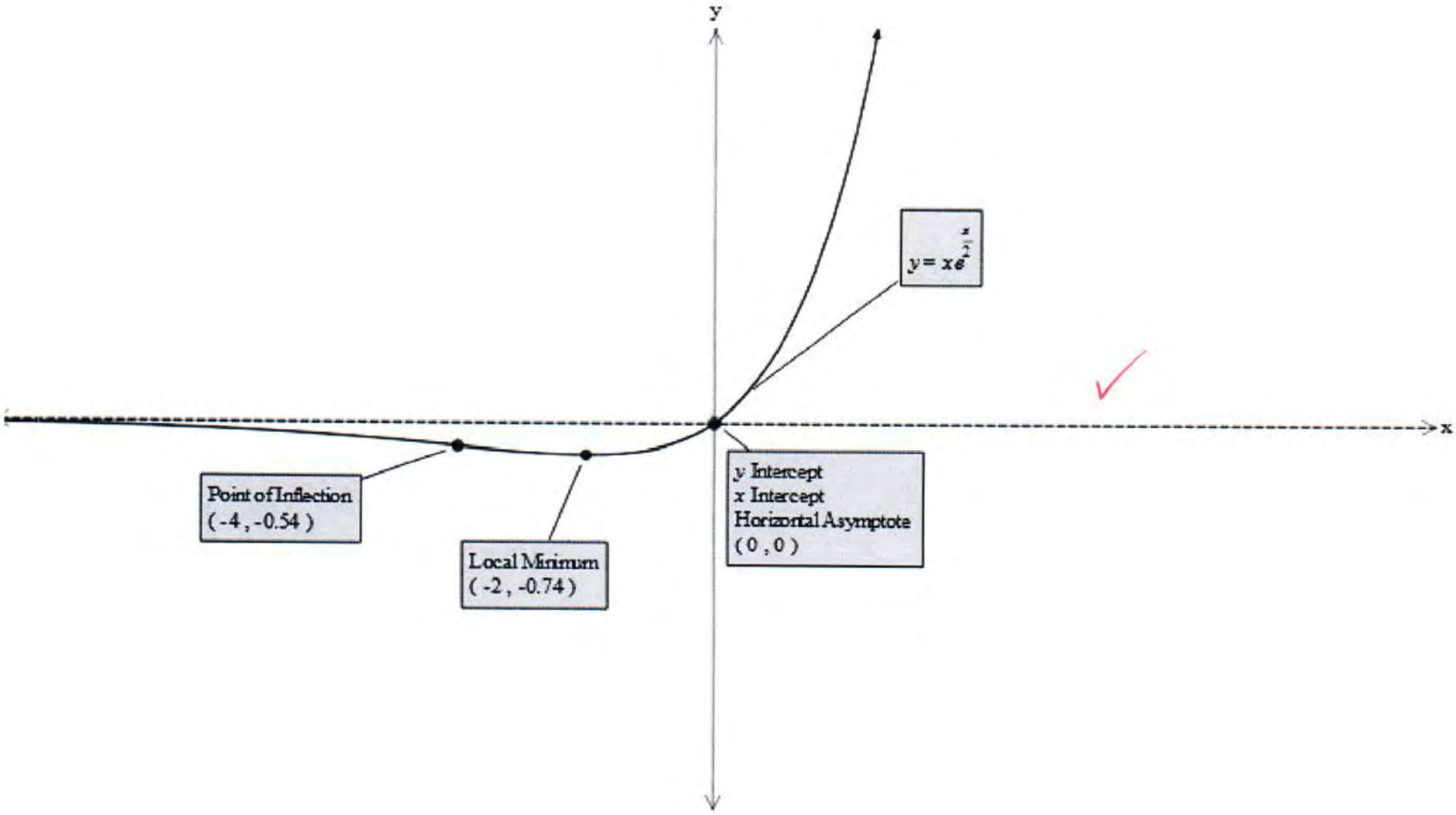
$y = xe^{x/2}$

$y' = e^{x/2}\left(\frac{x}{2}+1\right)$

$y'=0$, $x=-2$, $y=-2/e$ $\quad$ $-\frac{1}{2}$ if not shown min TP.

x	-3	-2	0
y'	$-$	0	$+$

_/ min at $(-2, -2/e)$ ✓



✓

$x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to 0$ $\therefore$ hort. asy at $y=0$ ✓

x-intercept $=0$, y-intercept $=0$

2U THSC 2016 Q14 solutions

Mean (out of 15): 8.84

(a) (i) $P(A \text{ wins on 1st draw})$
$= \frac{2}{5}$

0	1	Mean
1	184	0.995

(ii) $P(B \text{ wins on 1st draw})$
$= \frac{3}{5} \times \frac{3}{5}$
$= \frac{9}{25}$

0	1	Mean
102	83	0.449

A large number of students didn't understand that, for B to win, A had to lose first.

(iii) $P(A \text{ wins in fewer than 4 turns})$
$= P(\text{wins in } 1) + P(\text{wins in } 2) + P(\text{wins in } 3)$
$= \frac{2}{5} + \frac{3}{5} \times \frac{2}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{2}{5}$
$= \frac{2}{5} + \frac{12}{125} + \frac{72}{3125}$
$= \frac{1250 + 300 + 72}{3125}$
$= \frac{1622}{3125}$

0	0.5	1	1.5	2	Mean
95	3	6	16	65	0.873

Again, students didn't appreciate that A and B take it in turns.

(iv) $P(A \text{ wins})$
$= \frac{2}{5} + \frac{3}{5} \times \frac{2}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{2}{5}$
$+ \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} \times \frac{2}{5} + \ldots$
$= \dfrac{\frac{2}{5}}{1 - \frac{6}{25}}$
$= \dfrac{\frac{2}{5}}{\frac{19}{25}}$
$= \frac{10}{19}$

0	0.5	1	1.5	2	Mean
108	2	9	5	61	0.754

Similar problems to earlier parts.

(b) $y = x^3 + x^2$
$y' = 3x^2 + 2x$
$y'' = 6x + 2$

For point of inflexion $y'' = 0$
$\therefore 6x + 2 = 0$
$\therefore x = -\frac{1}{3}$

x	-1	$-\frac{1}{3}$	0
y''	-4	0	2
Concavity	Down	–	Up

Change of concavity
$\therefore$ Pt of inflexion at $\left(-\frac{1}{3}, \frac{2}{27}\right)$

0	0.5	1	1.5	2	Mean
5	27	64	26	63	1.311

Some students lost marks for not calculating actual values for y'' for points on either side of the possible point of inflexion to demonstrate that there has been a change of concavity. Some students used a y''' test to indicate that there has been a change of concavity. This is a legitimate, but not standard, technique. A number of students found and classified stationary points but this was not required.

(c) (i) $\frac{x}{6} = \frac{EH}{AH} = \frac{7}{5}$ (Proportional division theorem)
$\therefore x = \frac{42}{5}$
$= 8\frac{2}{5}$

0	0.5	1	1.5	2	Mean
22	4	29	19	111	1.522

A large number of students proved the result using similar triangles (the basis of the Proportional Division Theorem).

(ii) $\dfrac{BA}{8} = \dfrac{x+6}{x}$

$BA = \dfrac{\frac{42}{5}+6}{\frac{42}{5}} \times 8$

$= \dfrac{72}{42} \times 8$

$= 13\frac{5}{7}$

0	0.5	1	Mean
40	10	135	0.757

Most students were able to calculate the appropriate value. A common error involved students not realising that their calculation for BA required x+6 to be used, not just 6.

(d) Amt owing after 1 month

$= 500000 \times 1.01 - M$

... 2 months

$= 500000 \times 1.01^2 - M \times 1.01 - M$

... 300 months

$= 500000 \times 1.01^{300} - M.\ 1.\ \dfrac{(1.01^{300} - 1)}{1.01 - 1}$

$\dfrac{M(1.01^{300} - 1)}{0.01} = 500\,000 \times 1.01^{300}$

$M = \dfrac{500000 \times 1.01^{300} \times 0.01}{1.01^{300} - 1}$

$= \$5266.12$

Amt owing after 144 months

$= 500\,000 \times 1.01^{144} - 5266.12 \times \dfrac{1.01^{144} - 1}{0.01}$

$= 415\,091.36$

$\therefore$ the \$400000 prize will not clear the debt

0	0.5	1	1.5	2	2.5	3	Mean
35	19	14	10	17	12	78	1.819

Sadly some students did not realise that this question related to geometric series. Others, having calculated the monthly payment required to pay off the loan in 25 years did not know how to use it to calculate the amount owing after 12 years.

(ii) Amt after 1 month $= 15091.34 \times 1.01 - 5266.12$

$= 9976.13$

... 2 months $= 9976.13 \times 1.01 - 5266.12$

$= 4809.77$

... 3 months $= 4809.77 \times 1.01 - 5266.12$

$= -408.25$

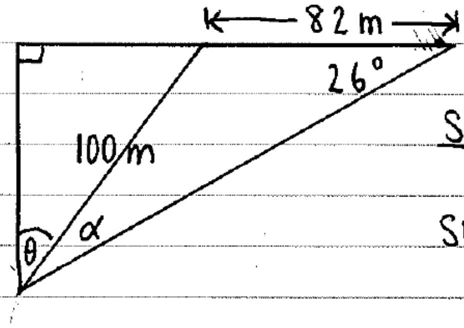
$\therefore$ Debt cleared after 3 months Refund \$408.25

0	0.5	1	Mean
97	41	47	0.365

A number of students assumed that the requirement to pay interest on the remaining \$15091.34 would not push the number of payments required beyond 3. They made their calculation by simply dividing \$15091.34 by \$5266.12. (If they had used this technique on the original debt of \$500000 they would have deduced that only 95 payments would be required instead of the actual 300 payments.)

Question 15

a)



$$\frac{\sin\alpha}{82} = \frac{\sin 26^\circ}{100}$$

$$\sin\alpha = \frac{82\sin 26^\circ}{100}$$

$$\alpha = \sin^{-1}\left(\frac{82\sin 26^\circ}{100}\right)$$

$$= 21^\circ\, 4'$$

Angle sum of a triangle:

$$26^\circ + 90 + 21^\circ + \theta = 180^\circ$$

$$137^\circ + \theta = 180^\circ$$

$$\theta = 43^\circ$$

Comments:

Majority of students answers this question very well.
Common mistakes were:

- that some students forgot to give there answer to the nearest degree,
- also some students calculated the wrong angle.

b) $f(x) = kx^2 - (3k-4)x + k$

positive definite when $\Delta < 0$ and $k > 0$.

$$\Delta = (3k-4)^2 - 4(k)(k)$$
$$= 9k^2 - 24k + 16 - 4k^2$$
$$= 5k^2 - 24k + 16$$

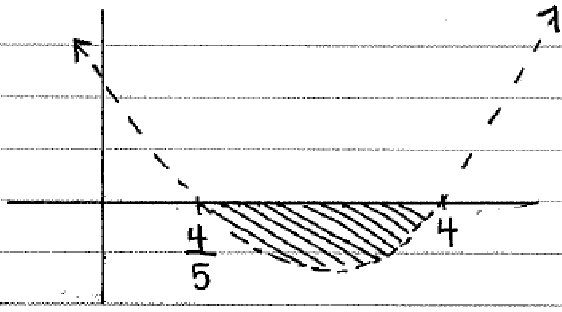
$\Delta = 0 \Rightarrow 5k^2 - 24k + 16 = 0$ $\therefore P = 80$, $S = -24$ $\Big\}$ $(-4, -20)$

$$\frac{(5k-20)(5k-4)}{5} = 0$$

$$\frac{\cancel{5}(k-4)(5k-4)}{\cancel{5}} = 0$$

$$\therefore k = 4, \frac{4}{5}$$

$\Delta < 0$:



$\therefore \frac{4}{5} < k < 4$

and $k > 0$.

Comments:

This question was very poorly answered. This question was a 'show that' question.
Common errors were:

- that some students didn't show where the discriminant was less than zero,
- also some students incorrectly calculated the discriminant.

c) i) $\left[d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}} \right]$

Equation of KO: $y - 2x = 0$

$PM = PN$

$ma = \frac{|2a - ma + 0|}{\sqrt{(2)^2 + (-1)^2}}$

$\therefore ma = \frac{|(2-m)a|}{\sqrt{5}}$

ii) From the diagram: $a > 0$ and $0 < m < 2$

$\therefore \sqrt{5}\, ma = |(2-m)a|$

$\sqrt{5}\, ma = 2a - ma$

$ma + \sqrt{5}\, ma = 2a$

$ma(1 + \sqrt{5}) = 2a$

$m(1 + \sqrt{5}) = 2$

$\therefore m = \frac{2}{1 + \sqrt{5}}$

iii) If $PN = \sqrt{5} - 1$,
$PM = \sqrt{5} - 1$.

$$\therefore \frac{PM}{MO} = \frac{2}{1+\sqrt{5}}$$

$$\frac{\sqrt{5}-1}{MO} = \frac{2}{1+\sqrt{5}}$$

$$MO = \frac{(1+\sqrt{5})(\sqrt{5}-1)}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

$$\therefore \ P \text{ is } \left(2, \frac{4}{1+\sqrt{5}}\right)$$

$$\left(2, \sqrt{5}-1\right)$$

Comments:

This question was very poorly answered.
Common mistakes were:

- that students didn't correctly 'show' what the value of *ma* was,
- students also produced the negative value of m not realising that the gradient of *m* is positive and between 0 and 2.

d) i) $\frac{d}{dx}(x^2+1)^3 = 3(x^2+1)^2 . 2x$

$$= 6x(x^2+1)^2$$

ii) $\int_0^1 5x(x^2+1)^2\,dx$

$= 5\int_0^1 x(x^2+1)^2\,dx$

$= \frac{5}{6}\int_0^1 6x(x^2+1)^2\,dx$

$= \frac{5}{6}\left[(x^2+1)^3\right]_0^1$

$= \frac{5}{6}\left[(1^2+1)^3 - (0^2+1)^3\right]$

$= \frac{5}{6}[7]$

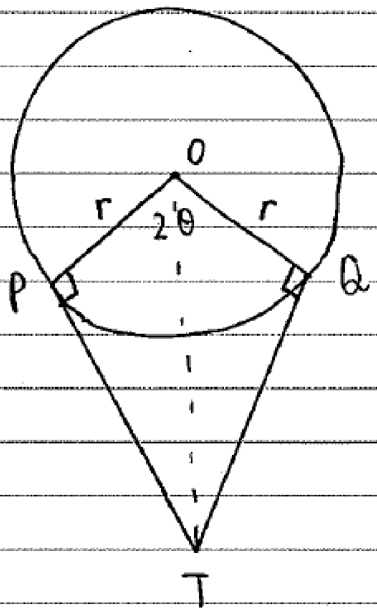
$= \frac{35}{6}$ $\left(5\ \frac{5}{6}\right)$

Comments:

Part (i), was answered well and part (ii) not so well.
Common mistakes were:

- that in part (i), students didn't multiply be the derivative of inside the brackets,
- in part (ii), students either didn't balance out the question so that you have the answer from part (i),
- also students made errors in substituting in the bounds of the integral.

e)



i) In Δ OPT : $\tan\theta = \dfrac{PT}{r}$

$\therefore\ PT = r\tan\theta$

ii) Area $= 2 \times \dfrac{1}{2} \times r \times r\tan\theta - \dfrac{1}{2} \times r^2 \times 2\theta$

$= r^2(\tan\theta - \theta)$

Comments:

Part (i), was answered well and part (ii) not so well.
Common mistakes were:

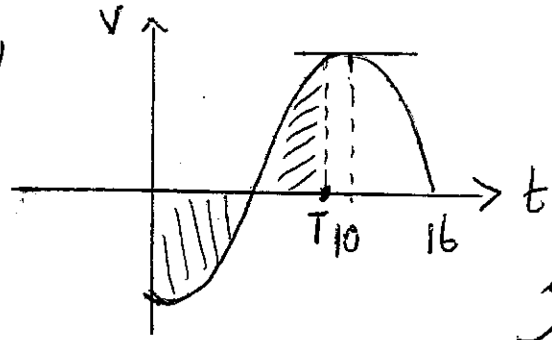
- students found the area of the sector, not the area of the shaded part.
- most students also just found the area of the segments

Trial HSC 2 unit 2016. 16 (1) (/15)

(16) (a) (i) when is the first time graph is above the x axis, v is positive ⇒ moving forward
$t > 4$ (1) Accept $4<t<16$

(ii) treat the graph as a curve sketching exercise. Accel positive means slope of tangent line to the (v) curve positive $0<t<10$ (1)

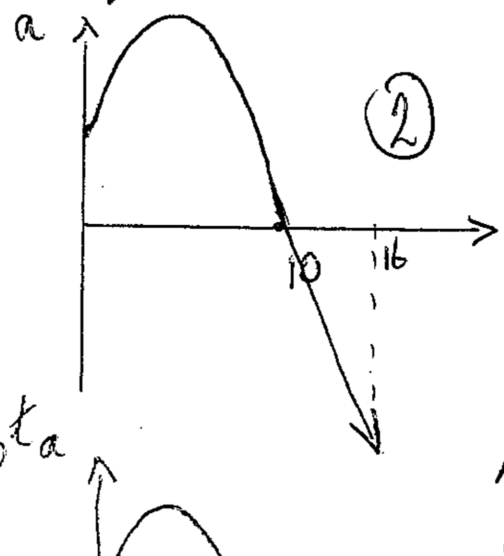
(iii)



2 shaded areas cancel each other out.
Area of unshaded part will provide answer to furthest away. $t=16$ (1)

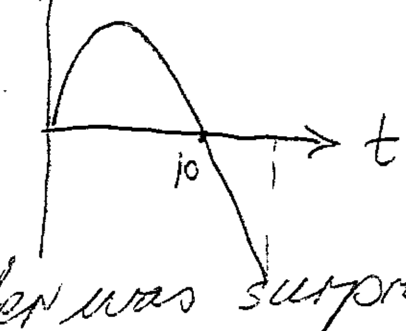
(iv) we need to work out T from my graph
Will accept $t = 7, 8, 9$ (1)

(v)



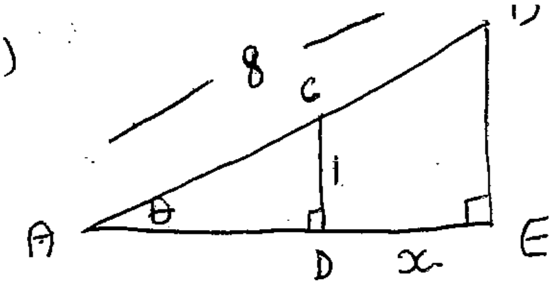
Positive slope ⇒ above x axis $0<t<10$.
Horizontal tangent $t=10$, slope 0.
$t>10$ curve has negative slope.
Prob inflexion point between min and max.

accept



Marker was surprised how well this question was answered. If you take the graph as a curve sketching / tangent etc exercise, most questions can be solved that way

(b) (i)



$\triangle ACD \parallel\!\mid \triangle ABE$ 2 angle test.

$$\frac{AC}{8} = \frac{1}{BE} = \frac{AD}{AD+x}$$

Get some info: $\tan\theta = \frac{1}{AD}$

$$\sin\theta = \frac{BE}{8} \Rightarrow BE = 8\sin\theta$$

$$\cos\theta = \frac{AD+x}{8} \Rightarrow AD + x = 8\cos\theta$$

$$\tan\theta = \frac{BE}{AE} = \frac{8\sin\theta}{AD+x} \quad *$$

Use $*$ $(AD+x)\tan\theta = 8\sin\theta$

we need to keep x so $\tan\theta = \frac{1}{AD}$ from above $\Rightarrow AD = \frac{1}{\tan\theta}$

$$\left(\frac{1}{\tan\theta} + x\right)\tan\theta = 8\sin\theta$$

$$1 + x\tan\theta = 8\sin\theta$$

$$x\tan\theta = 8\sin\theta - 1$$

$$x = \frac{8\sin\theta - 1}{\tan\theta} \quad (2)$$

OR $x = \frac{\cos\theta(8\sin\theta - 1)}{\sin\theta}$

OR $x = \cot\theta(8\sin\theta - 1)$

OR $= 8\cos\theta - \cot\theta$

Many forms that x can take. Well answered by most.

(ii) Given $x = \frac{8\sin\theta - 1}{\tan\theta}$

$$\dot{x} = \frac{\tan\theta \times 8\cos\theta - (8\sin\theta - 1) \times \sec^2\theta}{\tan^2\theta}$$

$$= \frac{8\cos\theta\tan\theta - 8\sin\theta\sec^2\theta + \sec^2\theta}{\tan^2\theta}$$

$$= \frac{8.C.\frac{S}{C} - 8.S.\frac{1}{C^2} + \frac{1}{C^2}}{t^2}$$

page 3

$$= \frac{8s - \frac{8s}{c^2} + \frac{1}{c^2}}{t^2}$$

$$= \frac{8sc^2 - 8s + 1}{c^2 t^2}$$

$$= \frac{8s(1-s^2) - 8s + 1}{c^2 \cdot \frac{s^2}{c^2}}$$

$$= \frac{8s - 8s^3 - 8s + 1}{s^2}$$

$$= \frac{1 - 8\sin^3\theta}{\sin^2\theta}$$

make $\dot{x} = 0 \Rightarrow 1 - 8\sin^3\theta = 0$

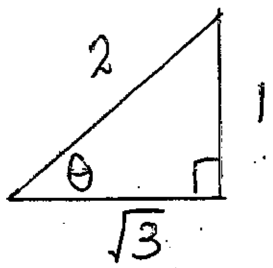
$$8\sin^3\theta = 1,$$

$$\sin^3\theta = \frac{1}{8}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ, \left(\frac{\pi}{6}\right)$$

$$\left(\frac{1}{8} < \sin\theta < 1 \text{ data}\right)$$

If $\sin\theta = \frac{1}{2}$



$$x = \frac{8\sin\theta - 1}{\tan\theta}$$

$$= \frac{8 \times \frac{1}{2} - 1}{\frac{1}{\sqrt{3}}} = 3\sqrt{3} \text{ m.}$$

check for max

$$\dot{x} = \frac{1 - 8s^3}{s^2}$$

$$\ddot{x} = \frac{s^2 \times -24s^2c - (1 - 8s^3) \times 2sc}{s^4}$$

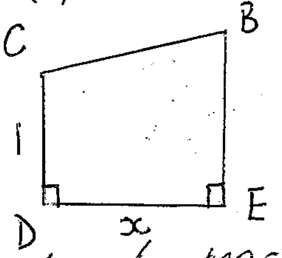
A lot to do here. Find θ, x and establish using $\ddot{x}$ or other means that we get a max. Not well answered.

Using the above triangle

$$\ddot{x} = \frac{\frac{1}{4} - 24\times\frac{1}{4}\times\frac{\sqrt{3}}{2} - \left(1-8\times\frac{1}{8}\right)\times 2\times\frac{1}{2}\times\frac{\sqrt{3}}{2}}{\frac{1}{16}}$$

$$= \left(\frac{1}{4} - 3\sqrt{3}\right)\times 16 < 0 \quad \text{maximum!}$$ (2)

(iii)



area trapezium

$$M = \frac{1}{2}(CD + BE)\times DE$$

$$= \frac{1}{2}(1 + 8\sin\theta)\times\left(\frac{8\sin\theta - 1}{\tan\theta}\right)$$

$$= \frac{1}{2}(1+8\sin\theta)\times\left(\frac{8\sin\theta}{\frac{\sin\theta}{\cos\theta}} - \frac{1}{\tan\theta}\right)$$

$$M = \frac{1}{2}(1+8\sin\theta)(8\cos\theta - \cot\theta)$$ (2)

Well done by most but was dependent on finding $BE = 8\sin\theta$ and writing x as $8\cos\theta - \cot\theta$.

(iv) From (ii) $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

test $\theta = \frac{\pi}{6}$ and $\frac{\pi}{4}$ into M of (iii)

$$x = \frac{8\sin\frac{\pi}{6} - 1}{\tan\frac{\pi}{6}} = \frac{3}{\frac{1}{\sqrt{3}}} = 3\sqrt{3} \quad (5.196\ldots)$$

and $$x = \frac{8\sin\frac{\pi}{4} - 1}{\tan\frac{\pi}{4}} = \frac{8\times\frac{1}{\sqrt{2}} - 1}{1} = 4\sqrt{2} - 1 \quad (4.656\ldots)$$

$$M = \frac{1}{2}\left(1+8\sin\frac{\pi}{6}\right)\left(8\cos\frac{\pi}{6} - \cot\frac{\pi}{6}\right)$$
$$= \left(\frac{1}{2}+4\times\frac{1}{2}\right)\left(8\times\frac{\sqrt{3}}{2} - \sqrt{3}\right) = (2.5)(3\sqrt{3}) \quad (12.99\ldots)$$
$$= \frac{15\sqrt{3}}{2}.$$

and $$M = \frac{1}{2}\left(1+8\sin\frac{\pi}{4}\right)\left(8\cos\frac{\pi}{4} - \cot\frac{\pi}{4}\right)$$
$$= \left(\frac{1}{2}+4\times\frac{1}{\sqrt{2}}\right)\left(8\times\frac{1}{\sqrt{2}} - 1\right) = \left(\frac{1}{2}+2\sqrt{2}\right)\left(4\sqrt{2}-1\right)$$
$$= 15\tfrac{1}{2}. \quad (15.498\ldots)$$

Now $\frac{15\sqrt{3}}{2} < 15\frac{1}{2}$.

From this we can see the value of M at $\theta = \frac{\pi}{6}$ is less than the value of M at $\theta = \frac{\pi}{4}$.

3

$\therefore$ M does not attain its maximum value when x attains its max. value.

Not well answered by nearly all students. Marked very liberally here!